Question	Scheme	Marks	AOs
1(a)(i)	$50x^2+38x+9 \equiv A(5x+2)(1-2x)+B(1-2x)+C(5x+2)^2$ $\Rightarrow B=\ldots$ or $C=\ldots$	M1	1.1b
	$B = 1$ and $C = 2$	A1	1.1b
(a)(ii)	E.g. $x = 0$ $x=0 \Rightarrow 9=2A+B+4C$ $\Rightarrow 9=2A+1+8 \Rightarrow A=\ldots$	M1	2.1
	$A = 0$*	A1*	1.1b
		(4)	
(b)(i)	$\frac{1}{(5x+2)^2}=(5x+2)^{-2}=2^{-2}\left(1+\frac{5}{2}x\right)^{-2}$ or $(5x+2)^{-2}=2^{-2}+\ldots$	M1	3.1a
	$\left(1+\frac{5}{2}x\right)^{-2}=1-2\left(\frac{5}{2}x\right)+\frac{-2(-2-1)}{2!}\left(\frac{5}{2}x\right)^2+\ldots$	M1	1.1b
	$2^{-2}\left(1+\frac{5}{2}x\right)^{-2}=\frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2+\ldots$	A1	1.1b
	$\frac{1}{(1-2x)}=(1-2x)^{-1}=1+2x+\frac{-1(-1-1)}{2!}(2x)^2+.$	M1	1.1b
	$\frac{1}{(5x+2)^2}+\frac{2}{1-2x}=\frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2+\ldots+2+4x+8x^2+\ldots$	dM1	2.1
	$=\frac{9}{4}+\frac{11}{4}x+\frac{203}{16}x^2+\ldots$	A1	1.1b
(b)(ii)	$\lvert x\rvert<\frac{2}{5}$	B1	2.2a
		(7)	
		(11 marks)	

Notes

(a)(i)

M1: **Uses a correct identity** and makes progress using an appropriate strategy (e.g. sub $x=\frac{1}{2}$) to find a value for B or C. May be implied by one correct value (cover up rule).

A1: Both values correct

(a)(ii)

M1: Uses an appropriate method to establish an equation connecting A with B and/or C and uses their values of B and/or C to find a suitable equation in A.

Amongst many different methods are:

Compare terms in x^2 $\Rightarrow 50=-10A+25C$ which would be implied by $50=-10A+25\times"2"$

Compare constant terms or substitute $x=0 \Rightarrow 9=2A+B+4C$ implied by $9=2A+1+4\times 2$

A1*: Fully correct proof with no errors.

Note: The second part is a proof so it is important that a suitable proof/show that is seen.

Candidates who write down 3 equations followed by three answers (with no working) will score M1 A1 M0 A0 (b)(i)

M1: Applies the key steps of writing $\frac{1}{(5x+2)^2}$ as $(5x+2)^{-2}$ and takes out a factor of 2^{-2} to form an expression of the form $(5x+2)^{-2} = 2^{-2}(1+*x)^{-2}$ where * is not 1 or 5

Alternatively uses direct expansion to obtain $2^{-2} + \dots$

M1: Correct attempt at the binomial expansion of $(1+*x)^{-2}$ up to the term in x^2

Look for $1+(-2)*x+\frac{(-2)(-3)}{2}*x^2$ where * is not 5 or 1.

Condone sign slips and lack of $*^2$ on term 3. ....

Alt Look for correct structure for 2nd and 3rd terms by direct expansion. See below

A1: For a fully correct expansion of $(2+5x)^{-2}$ which may be unsimplified. This may have been combined with their '*B*'

A direct expansion would look like $(2+5x)^{-2} = 2^{-2}+(-2)2^{-3}\times 5x+\frac{(-2)(-3)}{2}2^{-4}\times(5x)^2$

M1: Correct attempt at the binomial expansion of $(1-2x)^{-1}$

Look for $1+(-1)*x+\frac{(-1)(-2)}{2}*x^2$ where * is not 1

dM1: Fully correct strategy that is dependent on the previous **TWO** method marks.
There must be some attempt to use their values of *B* and *C*

A1: Correct expression or correct values for *p*, *q* and *r*.

(b)(ii)

B1: Correct range. Allow also other forms, for example $-\frac{2}{5}<x<\frac{2}{5}$ or $x\in\left(-\frac{2}{5},\frac{2}{5}\right)$

Do not allow multiple answers here. The correct answer must be chosen if two answers are offered

Question	Scheme	Marks	AOs
2(a)	$\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow A = ..., B = ...$	M1	1.1b
	Either $A = 2$ or $B = -1$	A1	1.1b
	$\frac{3}{(2x-1)(x+1)} = \frac{2}{2x-1} - \frac{1}{x+1}$	A1	1.1b
		(3)	
(b)	$\int \frac{1}{V} \mathrm{d}V = \int \frac{3}{(2t-1)(t+1)} \mathrm{d}t$	B1	1.1a
	$\int \frac{2}{2t-1} - \frac{1}{t+1} \mathrm{d}t = ...\ln(2t-1) - ...\ln(t+1)(+c)$	M1	3.1a
	$\ln V = \ln(2t-1) - \ln(t+1)(+c)$	A1ft	1.1b
	Substitutes $t = 2, V = 3 \Rightarrow c = (\ln 3)$	M1	3.4
	$\ln V = \ln(2t-1) - \ln(t+1) + \ln 3$ $V = \frac{3(2t-1)}{(t+1)}$ *	A1*	2.1
		(5)	
	(b) Alternative separation of variables:		
	$\int \frac{1}{3V} \mathrm{d}V = \int \frac{1}{(2t-1)(t+1)} \mathrm{d}t$	B1	1.1a
	$\frac{1}{3}\int \frac{2}{2t-1} - \frac{1}{t+1} \mathrm{d}t = ...\ln(2t-1) - ...\ln(t+1)(+c)$	M1	3.1a
	$\frac{1}{3}\ln 3V = \frac{1}{3}\ln(2t-1) - \frac{1}{3}\ln(t+1)(+c)$	A1ft	1.1b
	Substitutes $t = 2, V = 3 \Rightarrow c = \left(\frac{1}{3}\ln 3\right)$	M1	3.4
	$\frac{1}{3}\ln V = \frac{1}{3}\ln(2t-1) - \frac{1}{3}\ln(t+1) + \frac{1}{3}\ln 3$ $V = \frac{3(2t-1)}{(t+1)}$ *	A1*	2.1
		(5)	
(c)	(i) 30 (minutes)	B1	3.2a
	(ii) 6 (m^3)	B1	3.4
		(2)	
			(10 marks)
Notes:			

(a)

M1: Correct method of partial fractions leading to values for their A and B

E.g. substitution: $\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = A(x+1) + B(2x-1) \Rightarrow A = ..., B = ...$

Or compare coefficients $\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = x(A+2B) + A - B \Rightarrow A = ..., B = ...$

Note that $\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = A(2x-1) + B(x+1) \Rightarrow A = ..., B = ...$ scores M0

A1: Correct value for "A" or "B"

A1: Correct partial fractions not just values for "A" and "B". $\frac{2}{2x-1}-\frac{1}{x+1}$ or e.g. $\frac{2}{2x-1}+\frac{-1}{x+1}$

Must be seen as **fractions** but if not stated here, allow if the correct fractions appear later.

(b)

B1: Separates variables $\int \frac{1}{V}\mathrm{d}V = \int \frac{3}{(2t-1)(t+1)}\,\mathrm{d}t$. May be implied by later work.

Condone omission of the integral signs but the dV and dt must be in the correct positions if awarding this mark in isolation but they may be implied by subsequent work.

M1: Correct attempt at integration of the partial fractions.

Look for $...\ln(2t-1)+...\ln(t+1)$ where … are constants.

Condone missing brackets around the (2t – 1) and/or the (t + 1) for this mark

A1ft: Fully correct equation following through their A and B **only**.

No requirement for +c here.

The brackets around the (2t – 1) and/or the (t + 1) must be seen or implied for this mark

M1: Attempts to find "c" or e.g. "ln k" using $t=2,\ V=3$ following an attempt at integration.

Condone poor algebra as long as $t=2,\ V=3$ is used to find a value of their constant.

Note that the constant may be found immediately after integrating or e.g. after the ln's have been combined.

A1*: Correct processing leading to the given answer $V=\frac{3(2t-1)}{(t+1)}$

Alternative:

B1: Separates variables $\int \frac{1}{3V}\mathrm{d}V = \int \frac{1}{(2t-1)(t+1)}\,\mathrm{d}t$. May be implied by later work.

Condone omission of the integral signs but the dV and dt must be in the correct positions if awarding this mark in isolation but they may be implied by subsequent work.

M1: Correct attempt at integration of the partial fractions.

Look for $...\ln(2t-1)+...\ln(t+1)$ where … are constants.

Condone missing brackets around the (2t – 1) and/or the (t + 1) for this mark

A1ft: Fully correct equation following through their A and B **only**.

No requirement for +c here.

The brackets around the (2t – 1) and/or the (t + 1) must be seen or implied for this mark

M1: Attempts to find "c" or e.g. "ln k" using $t=2,\ V=3$ following an attempt at integration.

Condone poor algebra as long as $t=2,\ V=3$ is used to find a value of their constant.

Note that the constant may be found immediately after integrating or e.g. after the ln's have been combined.

A1*: Correct processing leading to the given answer $V=\frac{3(2t-1)}{(t+1)}$

(Note the working may look like this:

$$\frac{1}{3}\ln 3V=\frac{1}{3}\ln(2t-1)-\frac{1}{3}\ln(t+1)+c,\ \frac{1}{3}\ln 9=\frac{1}{3}\ln(3)-\frac{1}{3}\ln 3+c,\ \ c=\frac{1}{3}\ln 9$$

$$\ln 3V=\ln\frac{9(2t-1)}{(t+1)}\Rightarrow 3V=\frac{9(2t-1)}{(t+1)}\Rightarrow V=\frac{3(2t-1)}{(t+1)}*)$$

Note that B0M1A1M1A1 is not possible in (b) as the B1 must be implied if all the other marks have been awarded.

Note also that some candidates may use different variables in (b) e.g.

$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3y}{(2x-1)(x+1)}\Rightarrow \int \frac{1}{y}\mathrm{d}y=\int \frac{3}{(2x-1)(x+1)}\,\mathrm{d}x$ etc. In such cases you should award marks for equivalent work but they must revert to the given variables at the end to score the final mark.

Also if e.g. a "t" becomes an "x" within their working but is recovered allow full marks.

(c)

B1: Deduces 30 minutes. Units not required so just look for 30 but allow equivalents e.g. ½ an hour.
If units are given they must be correct so do not allow e.g. 30 hours.

B1: Deduces 6 m^3. Units not required so just look for 6. Condone $V < 6$ or $V \leq 6$
If units are given they must be correct so do not allow e.g. 6 m.

Question	Scheme	Marks	AOs
3(a)	e.g. $\dfrac{3kx-18}{(x+4)(x-2)} \equiv \dfrac{A}{x+4}+\dfrac{B}{x-2} \Rightarrow 3kx-18 \equiv A(x-2)+B(x+4)$ **or** $\dfrac{3kx-18}{(x+4)(x-2)} \equiv \dfrac{A}{x-2}+\dfrac{B}{x+4} \Rightarrow 3kx-18 \equiv A(x+4)+B(x-2)$	B1	1.1b
	$6k-18=6B \Rightarrow B=...$ or $-12k-18=-6A \Rightarrow A=...$ **or** $3kx-18 \equiv (A+B)x+4B-2A \Rightarrow A+B=3k,\ -18=4B-2A$ $\Rightarrow A=...$ or $B=...$	M1	1.1b
	$\dfrac{2k+3}{x+4}+\dfrac{k-3}{x-2}$	A1	1.1b
		(3)	
(b)	$\displaystyle\int \left(\frac{"2k+3"}{x+4}+\frac{"k-3"}{x-2}\right) dx = ...\ln(x+4)+...\ln(x-2)$	M1	1.2
	$("2k+3")\ln(x+4)+("k-3")\ln(x-2)$	A1ft	1.1b
	$("2k+3")\ln(5)-("k-3")\ln(5) \Rightarrow ("k+6")\ln 5 = 21 \Rightarrow k=...$	dM1	3.1a
	$(k=)\dfrac{21}{\ln 5}-6$	A1	2.2a
		(4)	
			(7 marks)

Notes

(a)

B1: Correct form for the partial fractions and sets up the correct corresponding identity which may be implied by two equations in A and B if they are comparing coefficients.

M1: Either

- substitutes $x=2$ or $x=-4$ in an attempt to find A or B in terms of k
- expands the rhs, collects terms and compares coefficients in an attempt to find A or B in terms of k

Or may be implied by one correct fraction (numerator **and** denominator)

You may see candidates substituting two other values of x and then solving simultaneous equations.

A1: Achieves $\dfrac{2k+3}{x+4}+\dfrac{k-3}{x-2}$ with no errors. Must be the correct partial fractions not just for correct numerators. May be seen in (b). Correct answer implies B1M1A1. One correct fraction only B0M1A0

(b)

M1: Attempts to find $\int \left(\frac{"2k+3"}{x+4} + \frac{"k-3"}{x-2} \right) \mathrm{d}x$. Score for either $\frac{\dots}{x+4} \to \dots\ln(x+4)$ or $\frac{\dots}{x-2} \to \dots\ln(x-2)$

Allow the … to be in terms of k or just constants but there must be no x terms.

Condone invisible brackets for this mark.

A1ft: $("2k+3")\ln|x+4| + ("k-3")\ln|x-2|$

but condone round brackets e.g. $("2k+3")\ln(x+4) + ("k-3")\ln(x-2)$ or equivalent e.g. $("2k+3")\ln(x+4) + ("k-3")\ln(2-x)$

Follow through their partial fractions with numerators which must both be in terms of k.

Condone missing brackets as long as they are recovered later e.g. when applying limits.

dM1: A full attempt to find the value of k. To score this mark they must have attempted to integrate their partial fractions, substituted in the correct limits, subtracted either way round, set = 21 and attempted to solve to find k. Condone omission of the terms containing ln(1) or ln(-1).

Note that e.g. ln(-5) or ln(5) must be seen but may be disregarded **after** substitution and subtraction.

Do not be concerned with the processing as long as they proceed to $k = \dots$

Condone if they use x instead of k after limits have been used as long as the intention is clear.

A1: Deduces $(k=)\frac{21}{\ln 5} - 6$ or exact equivalent e.g. $\frac{21 - 6\ln 5}{\ln 5}, \frac{21 - 3\ln 25}{\ln 5}$.

Allow recovery from expressions that contain e.g. $\ln(-5)$ as long as it is dealt with subsequently.

Also allow recovery from invisible brackets. Condone $x = \frac{21}{\ln 5} - 6$

Some candidates may use substitution in part (b) e.g.

$$\int \left(\frac{"2k+3"}{x+4} + \frac{"k-3"}{x-2} \right) \mathrm{d}x = \int \left(\frac{"2k+3"}{x+4} \right) \mathrm{d}x + \int \left(\frac{"k-3"}{x-2} \right) \mathrm{d}x$$

$$u = x+4 \Rightarrow \int \left(\frac{"2k+3"}{x+4} \right) \mathrm{d}x = \int \left(\frac{"2k+3"}{u} \right) \mathrm{d}u = \dots\ln u$$

$$u = x-2 \Rightarrow \int \left(\frac{"k-3"}{x-2} \right) \mathrm{d}x = \int \left(\frac{"k-3"}{u} \right) \mathrm{d}u = \dots\ln u$$

Score **M1** for integrating at least once to an appropriate form as in the main scheme e.g. …ln$u$

A1ft: For $("2k+3")\ln|u| + ("k-3")\ln|u|$

but condone $("2k+3")\ln u + ("k-3")\ln u$ which may be seen separately

Follow through their "A" and "B" in terms of k.

Condone missing brackets as long as they are recovered later e.g. when applying limits.

dM1: A full attempt to find the value of k. To score this mark they must have attempted to integrate their partial fractions using substitution, substituted in the correct changed limits and subtracts either way round, set = 21 and attempted to solve to find k. Do not be concerned with processing as long as they proceed to $k = \ldots$ Condone omission of terms which contain e.g. ln(1) or ln(-1).

Note that e.g. ln(-5) or ln(5) must be seen but may be disregarded **after** substitution and subtraction.

$$\left[(2k+3)\ln u\right]_1^5 + \left[(k-3)\ln u\right]_{-5}^{-1} = 21 \Rightarrow (2k+3)\ln 5 - (2k+3)\ln 1 + (k-3)\ln 1 - (k-3)\ln 5 = 21$$

$$\Rightarrow (2k+3)\ln 5 - (k-3)\ln 5 = 21 \Rightarrow (k+6)\ln 5 = 21 \Rightarrow k = \ldots$$

A1: $k = \dfrac{21}{\ln 5} - 6$ or exact equivalent e.g. $\dfrac{21-6\ln 5}{\ln 5}$, $\dfrac{21-3\ln 25}{\ln 5}$, $21\log_5 e - 6$.

Allow recovery from expressions that contain e.g. $\ln(-5)$ as long as it is dealt with subsequently.

Also allow recovery from invisible brackets.